An Effective Mathematics Program

EFFECTIVE INSTRUCTION IN MATHEMATICS, K-12

"In order for students to achieve excellence in an area like mathematics, there must be a balance between understanding basic math concepts, practising skills like multiplication tables, and developing the thinking skills needed for advanced problem solving. These foundational skills remain a focus—and combined with creativity and critical thinking, innovative problem solving, effective communication and collaboration, they lead to excellence."

Achieving Excellence: A Renewed Vision for Education in Ontario, April 2014, p. 5

Students learn best when they are interested and excited about what they are doing. In all mathematics classrooms, a diverse range of interests, learning preferences, and needs exist. Therefore, an effective mathematics program must include a wide range of instructional approaches that is responsive to evidence of student thinking. Our students cannot simply be passive observers, but instead must be active participants in their learning. Students must recognize that mathematics is not just a set of rules and procedures to be memorized without a deep understanding of the relationships that form the foundation of those rules. An effective mathematics program must help students make sense of their surroundings and see relationships in everything they do. Linking math with other subject areas and daily life helps us to recognize how important mathematics is in the world outside of school.

Beliefs for an Effective Mathematics Program

- All students will achieve success
- Instruction is based on the evidence of sound research that has been verified by classroom practice
- All students will develop a fundamental understanding of mathematical concepts
- A positive attitude and classroom environment is the foundation of life-long learning in mathematics

-Adapted from A Guide to Effective Instruction in Mathematics,, Volume 1, p. viii

Benefits of an Effective Mathematics Program

- Provides opportunities for students at all levels to practise and apply skills and procedures
- Strengthens the connections between skills, concepts, strategies and problem solving
- Addresses various interests, learning preferences and needs
- Gives learners a voice and an opportunity to express what they know and how they know it
- Builds accuracy, efficiency and flexibility of thinking

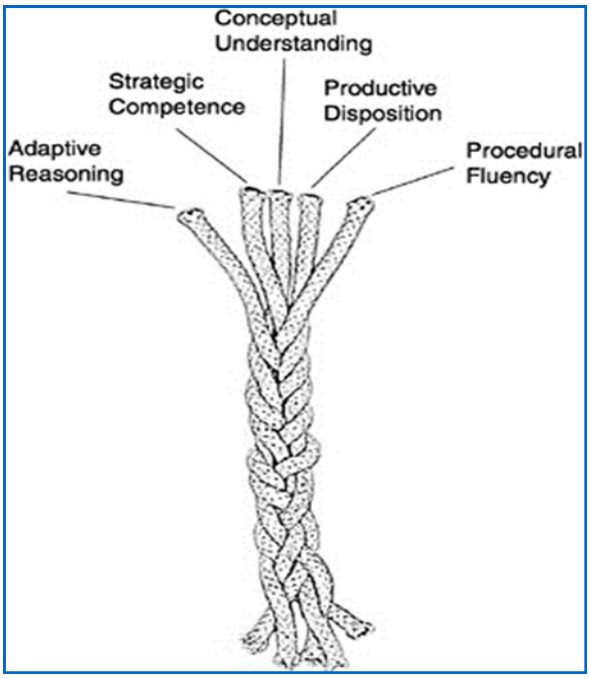
Proficiency in Mathematics

"Today's mathematics curricula and instruction must focus on preparing students to be mathematically proficient and compute accurately, efficiently, and flexibly. What does it mean to compute with accuracy, efficiency, and flexibility? **Accuracy** *denotes the ability to produce an accurate answer;* **efficiency** *refers to the ability to choose an appropriate, expedient strategy for a specific computation problem; and* **flexibility** *means the ability to use number relationships with ease in computation."*

Parrish, 2010

Intertwined Components of Proficiency

Mathematical proficiency has five components. These components are not independent: they represent different aspects of a complex whole.



These five components are interwoven and interdependent in the development of mathematical proficiency.

CONCEPTUAL UNDERSTANDING—comprehension of mathematical concepts, operations and relations

PROCEDURAL FLUENCY—skill in carrying out procedures flexibly, accurately, efficiently and appropriately

STRATEGIC COMPETENCE—ability to formulate, represent, and solve mathematical problems

ADAPTIVE REASONING—capacity for logical thought, reflection, explanation, and justification

PRODUCTIVE DISPOSITION—habitual inclination to see mathematics as sensible, useful and worthwhile, coupled with a belief in diligence and one's own efficacy

National Research Council (2001)
Adding it up: Helping children learn mathematics
Washington, CD. National Academy Press

Planning Considerations

"A problem-solving approach encourages students to reason their way to a solution or a new understanding. As students engage in reasoning, teachers further encourage them to justify solutions...this opens the door to recognizing the range of strategies that can be used to arrive at a solution. By seeing how others solve a problem, students can begin to think about their own thinking and the thinking of others, and to consciously adjust their strategies in order to make their solutions as efficient and accurate as possible."

The Ontario Curriculum, Grades 9 and 10 Mathematics, p. 10

Purposeful planning provides meaningful opportunities for students to learn <u>through</u> problem-solving in order to make sense of math. It is equally important for students to learn <u>about</u> problem-solving so that they can learn specific strategies or how to use tools to support their learning.

ADSB Planning Considerations for Mathematics Template

Teaching Through Problem Solving:
Explore, develop, and apply a conceptual understanding of a mathematical concept; teachers move away from the "watch me" method of teaching towards active participation for deeper understanding

May occur simultaneously

Teaching About Problem Solving:
Guide students through the development of inquiry or problem-solving processes and strategies

Planning With the Big Ideas in Mind

The big ideas are the essential concepts of focus within a unit. They are like an internal map, and allow students to see how topics and expectations connect (including the seven mathematical process expectations). Effective planning clusters expectations around a big idea so that teachers and students understand the purpose behind a task or lesson. They help us to understand where students are going and what will need to be done to get them there. Big ideas help to build connections between students' new and prior knowledge, and promote discussion of mathematical thinking. Big ideas are fundamentally simple and allow teachers to easily sequence and connect key concepts across grades. When teachers plan with the big ideas in mind, they build their mathematical knowledge for teaching.

For more information on Big Ideas, please see "*The Big Ideas from Dr. Small, K-3, 4-8 and 9-12*".

Supporting Student Learning in Mathematics

"Students in a mathematics class typically demonstrate diversity in the ways they learn best. It is important, therefore, that students have opportunities to learn in a variety of ways."

The Ontario Curriculum, Grades 9 and 10 Mathematics, p. 23

Instruction that is personalized and precise:

- **establishes clear mathematical learning goals and allows for the co-construction of success criteria**
- **builds on previous knowledge and learning**
- **anticipates a range of student mathematical thinking**
- **utilizes a variety of resources, materials and tools in thoughtful ways**
- **provides time and space for students to construct their understanding and engage in purposeful practice of the skills and concepts learned**
- **allows students to share their thinking and learning so that mathematical goals are achieved**
- **is continually adjusted based on information gathered on a day-by-day, minute-by-minute basis**

Differentiation

National Council of Teachers of Mathematics (NCTM) recognizes the need for accommodating differences among students, taking into account prior knowledge and intellectual strengths, to ensure that each student can learn important mathematics. *"Equity does not mean that every student should receive identical instruction; instead, it demands that reasonable and appropriate accommodations be made as needed to promote access and attainment for all students"* *(NCTM, 2000, p. 12).*

"Differentiating instruction means that some students may require remediation or modified expectations; others may need extension or opportunities for independent study" *(Learning for All, 2011).*

Instruction that is accessible and yet challenging for students leverages learner strengths while supporting individual needs. Multiple entry points allow for the engagement of all students. Student learning is continually monitored so that self, peer and teacher feedback can occur.

Effective Instruction in Mathematics

Instructional strategies and approaches that teachers employ will vary according to both the mathematical goals and the needs of the students. When students learn independently, cooperatively, with teacher direction and through hands-on experiences, they develop a solid understanding of mathematical concepts.

Adapted from The Ontario Curriculum, Grades 1-8 Mathematics, p. 24

Instructional Approaches

Provide a balance of approaches, dependent upon need and instructional purpose.

Shared Mathematics

- provides opportunities for students to learn from one another;
- encourages discussion and sharing of ideas;
- involves students working collaboratively to solve a problem or investigate a mathematical idea;
- can occur between pairs or in small groups of students, or it can involve large-group discussions;
- promotes the development of students' understanding of mathematical concepts and skills, to encourage problem solving, mathematical reasoning, as well as vocabulary and explanation/ reasoning skills.

Guided Mathematics

- reinforces a specific skill or concept;
- introduces the new skills or concepts required to solve a problem;
- introduces a specific process (e.g., a new problem-solving strategy; a particular algorithm for students to use);
- teaches specific conventions, such as fraction and decimal notation;
- models mathematical language, mathematical thinking, and problem solving;
- demonstrates flexible grouping.

Independent Mathematics

- provides opportunities for students to develop, consolidate, or apply strategies or skills on their own;
- provides opportunities for students to make choices independently;
- allows students to work at their own pace and to develop independence, perseverance, and self-confidence;
- gives students opportunities to demonstrate what they know and what they can do.

Mathematical Language and Word Study

- enables students to comprehend math vocabulary, terminology, texts and questions with increasing complexity;
- supports fluent mathematical reading and approaches;
- develops schema so that students understand what terminology requires the use or application of specific strategies.

Shared, Guided and Independent Instructional approaches as listed and described from ***A Guide to Effective Instruction in Mathematics, K - 6—Volume 1***

Questions for Consideration

When designing an effective mathematics program, teachers are encouraged to consider the following questions:

Mathematical Goals

- What curriculum expectations are the focus?
- How will I organize and present the big ideas?
- What do I want my students to know, do and understand at the end of this task?
- What will success look like for this task?
- How will I facilitate discussion so that learning goals are met?
- What will I see and hear that lets me know that students understand?

Task Selection

- What task would best help students to learn the concept or to develop the processes/ strategies?
- Will the task be meaningful for all students?
- How does the task build on previous learning?
- Am I providing a variety of learning opportunities for student engagement and understanding (e.g. journaling, games, fact practice, problem solving, conferencing, etc.)?
- Is there an opportunity for shared, guided and independent math?
- Have I included a plan for assessment?

Anticipating Needs

- Have I completed the task ahead of time to anticipate possible student questions or clarifications?
- Have I provided appropriate accommodations for students who require them?
- What are some ways the problem may be solved? What strategies might be used?
- How will I respond to misconceptions and move thinking forward?
- When and how will feedback be given and by whom?

Consolidating the Learning

- Which method of consolidation will I use (e.g. bansho, congress, gallery walk, etc.)?
- How will I order the presentation of solutions to help develop students' understanding of the mathematical ideas?
- How can I develop a math-talk community where students are questioning, justifying, making connections, generalizing, etc.?
- How will I decide teacher next steps for planning and instruction?

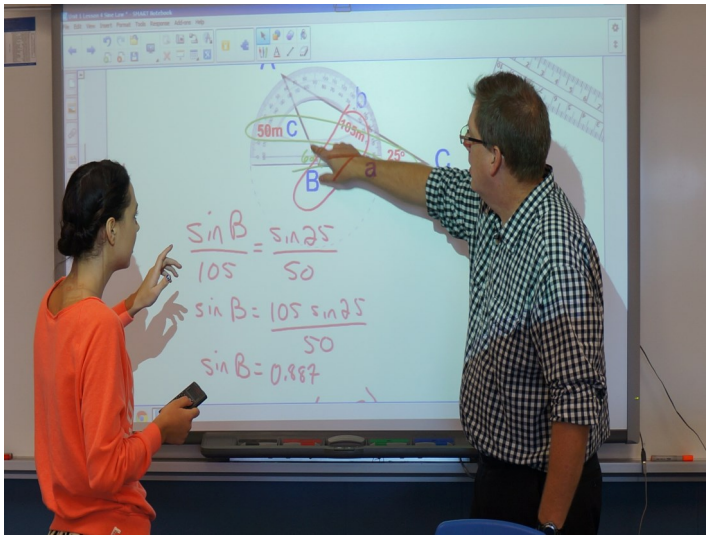
Deep Learning Tasks: How will I ensure that I have incorporated deep learning tasks which involve discovering and mastering existing knowledge and then creating and using new knowledge in the world?

Adapted from A Rich Seam, by Michael Fullan and Maria Langworthy

Key Questioning

Skillful questioning helps to evoke student thinking and build understanding of the math concepts taught. In order to know what questions to ask to move student thinking and understanding forward, it is critical that teachers continually work to develop their own knowledge of the mathematics. By developing their own understanding and by solving a problem before the lesson, and keeping the learning goal and the big ideas in mind, teachers can better anticipate possible student misconceptions and develop the scaffolding questions necessary to minimize these difficulties.

Adapted from the ADSB Planning Considerations for Mathematics Template and Asking Effective Questions, Capacity Building Series, July 2011



Building Mathematical Knowledge for Teaching

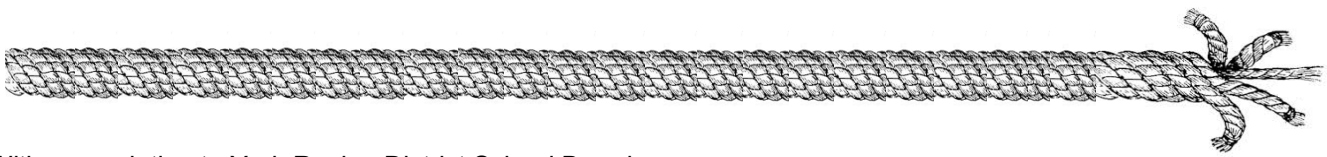
"Teachers and students who believe they will be successful set higher goals for themselves, try harder to achieve those goals, and persist through obstacles. Efficacy is a precursor to student achievement."

Dr. Catherine Bruce

The current approaches to teaching mathematics are very different from what many of us experienced as students ourselves. Today our roles are interrelated: activator, questioner, listener, and co-learner. These roles can be seen both inside and outside the classroom, as we engage in professional learning to build our mathematics knowledge for teaching and explore effective instructional practices. Developing our own efficacy as teachers of mathematics benefits everyone and leads to greater student efficacy and achievement as well.

Suggested Reading

- ***The Ontario Curriculum, Grades 1-8 Mathematics***, Ministry of Education (2005)
- ***The Ontario Curriculum, Grades 9 and 10 Mathematics,*** Ministry of Education (2005)
- ***The Ontario Curriculum, Grades 11 and 12 Mathematics,*** Ministry of Education (2007)
- ***The Guides to Effective Instruction in Mathematics K-3,*** Ministry of Education (2005/2007)
- ***The Guides to Effective Instruction in Mathematics Grades 4-6***, Ministry of Education (2006/2008)
- ***Teaching and Learning Mathematics: The Report of the Expert Panel on Mathematics in Grades 4 to 6 in Ontario***, Ministry of Education (2004)
- ***Early Math Strategy: The Report of the Expert Panel on Early Math in Ontario***, Ministry of Education (2003)
- ***Leading Math Success: The Report of the Expert Panel on Student Success in Ontario,*** Ministry of Education (2004)
- ***Learning for All,*** Ministry of Education (2013)
- ***TIPS4RM,*** Ministry of Education (2005)
- ***Knowing What Counts: Setting and Using Criteria***, Second Edition, Gregory et all (2011)
- ***Good Questions: Great Ways to Differentiate Mathematics Instruction,*** Marian Small (2009)
- ***Big Ideas from Dr. Small: K-3, Grades 4-8, Grades 9-12,*** Marian Small (2009 and 2010)
- ***Making Math Meaningful for Canadian Students K-8,*** Marian Small (2008)
- ***Eyes on Math: A Visual Approach to Teaching Math Concepts,*** Marian Small (2013)
- ***Number Talks: Helping Children Build Mental Math and Computation Strategies,*** Sherry Parrish (2010)
- **About Teaching Mathematics: A K-8 Resource**, Marilyn Burns (2009)
- **50 Problem-Solving Lessons**, Marilyn Burns (1996)
- **A Collection of Math Lessons Grades 1-3, 3-6 and 6-8**, Marilyn Burns (1991, 1993, 1995)
- **Teaching Student-Centered Mathematics PreK-2, Grades 3-5, Grades 6-8**, John Van de Walle (2014)
- **edugains.ca**
- **onetwoinfinity.ca** - Marian Small
- **ADSB Number Sense and Numeration Integration Document: Grades 3-8**
- **Planning Considerations For Mathematics** template—ADSB Sharepoint
- **Daily Lesson Template**—ADSB Sharepoint
- **ADSB Number Sense and Numeration Integration Document: Grades 3-8**
- **The Three-Part Problem Solving Mathematics** Smartcard



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